Reflection and refraction process of spinwave in a ferromagnet/frustrated ferromagnet junction system

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Frustration introduces a nontrivial dispersion relation of spinwave even in a ferromagnetic phase in a spin system. We study the reflection and refraction process of spinwaves in the ferromagnet/frustrated ferromagnet junction system by using the Holstein-Primakoff spinwave expansion and taking the large-S limit. We discuss the relation between the incident angle and refraction angle of spinwave, namely, the Snell's low of spinwave. As concrete examples of frustrated ferromagnets, we study the fully polarized ferromagnet phases in the J_1-J_2 chains and the J_1-J_2 model on the square lattice. The interesting refraction processes, such as the splitting of the incident spinwave and the negative refraction, are discussed. We also study the transmittance and reflectance in these concrete models.

KEYWORDS: spinwave, magnonics, junction system, frustration, negative refraction

1. Introduction

Spinwave is a collective excitation of spins in magnets. Since spinwave can carry information on the nano/microscale, recently, potential application of spinwave to new devices has attracted much attention to researchers.[1] In many circumstances, spinwave behaves like a sound or light wave; reflection and refraction process of spinwave can be considered.

Understanding of the reflection and refraction process is important to control spinwave. Theoretically, this process of junction system of usual ferromagnets whose spinwave has a trivial dispersion relation is understood by using geometrical-optics approximation[2-5] or Landau-Lifshitz-Gilbert equation.[6] One of the important results of these studies is that the direction of the propagating spinwave depends on the strength of applied magnetic field. The experimental control of propagation of spinwave is indeed realized by tuning the internal demagnetizing fields in a permalloy waveguid.[7] If the dispersion relation of spinwave is nontrivial, an exotic refraction process is expected. In the case of light, for example, the negative refraction occurs due to the nontrivial dispersion relation.[8, 9] Recently, it is theoretically proposed that the anisotropic nature of the dispersion relation of dipole-exchange spinwaves can result in the negative refraction.[6]

Recent experimental advance has been exhibiting a wide variety of frustrated magnets. Magnetic frustration provides us with an exotic magnetic behavior.[10] Even in a ferromagnet, in which all spins align, the frustration can induce an unusual dispersion relation of spinwaves. Hence, it will be natural to expect interesting refraction processes of spinwaves in frustrated ferromagnets. In this paper, we study the reflection and refraction process of spinwave in ferromagnet/frustrated ferromagnet junction system.

As a concrete example of frustrated ferromagnets, we consider the systems which are already realized experimentally. For example, frustrated ferromagnets can be prepared by applying high magnetic field on frustrated magnets. One-dimensional (1D) J_1-J_2 Heisenberg chain with a nearest-neighbor exchange coupling J_1 and next-nearest-neighbor coupling J_2 is one of the famous frustrated magnets.[11-15] In the fully polarized phase under high magnetic field, the dispersion of spinwave has two minima at[11, 16-18]

the wavevectors $\mathbf{Q}$s of $\cos \mathbf{Q} = -J_1/4J_2$ for $|J_1|/J_2 < 4$ and $J_2 > 0$. There are many compounds consisting of the J_1-J_2 chains (see Table. I in Ref. 12). The saturated ferromagnetic phases under high magnetic field are experimentally realized in[12] $Rb_2Cu_2Mo_3O_{12}$,[19-21] $(N_2H_5)CuCl_3$ and[22-24] $LiCuVO_4$.

As another example of the frustrated magnets, the various compounds of the square-lattice J_1-J_2 model, e.g., $BaCdVO(PO_4)_2$, are reported.[25, 26] Without external field, the collinear antiferromagnet phase appears in $BaCdVO(PO_4)_2$; the saturation field is about 4~6T. In the saturated ferromagnetic phase of this compound, theoretically, the dispersion relation is expected to be nontrivial; it has two minima at $\mathbf{Q} = (0, \pi)$ and $\mathbf{Q} = (\pi, 0)$.[25-27]

The organization of the present paper is as follows. In Sec. 2, we briefly review the dispersion relations of spinwaves in (frustrated) ferromagnets by using the Holstein-Primakoff spiwave expansion. In Sec. 3, we generally discuss refraction angles of transmission spinwaves in the ferromagnets junction systems satisfying the given conditions. Then, we apply this discussion to the several junction systems including frustrated J_1-J_2 chains or J_1-J_2 model on the square lattice. We shall explicitly see the splitting of the incident spinwave or the negative refraction in these models. In Sec. 4, we study the reflection- and transmission rate (reflectance and transmittance) in some of the junction systems considered in Sec. 3 by solving the simple Schröedinger equations within the large S limit. We also study the phase shift of the spinwave at the boundary. Our approach naturally treats the lattice structure of ferromagnets without the coarse graining.

2. Dispersion relation of Frustrated ferromagnets

As a brief review, let us discuss the dispersion relation of spinwaves in the fully-polarized ferromagnetic phase of (frustrated) spin systems. For simplicity, we consider the lattice systems with one magnetic ion per unit cell, and assume the rotational symmetry around the z direction in spin space. This assumption leads to the conservation law of the total angular momentum along the z direction, namely, $\sum_\mathbf{i} \langle S_\mathbf{i}^z \rangle$. We study the Hamiltonian with the generic exchange interactions $J_{\mathbf{ij}} = J_{\mathbf{i-j}} = J_{\mathbf{j-i}} = J_\mathbf{l}$, the on-site anisotropic interaction term

K, and the external magnetic field H:

$$H = \sum_{\langle i,j \rangle} J_{ij} \mathbf{S_i} \cdot \mathbf{S_j} + \sum_i (-K(S_i^z)^2 + H S_i^z) , \quad (1)$$

where we use the coordinate (x, y, z) in the spin space, and (a, b, c) in the lattice space. Let us rewrite the spin operator by using the Holstein-Primakoff transformation on the fully-polarized ferromagnetic phase:

$$S_i^z = -S + \alpha_i^\dagger \alpha_i ,$$

$$S_i^+ = \sqrt{2S} \alpha_i^\dagger \sqrt{1 - \frac{\alpha_i^\dagger \alpha_i}{2S}} \approx \sqrt{2S} \alpha_i^\dagger ,$$

$$S_i^- = \sqrt{2S} \sqrt{1 - \frac{\alpha_i^\dagger \alpha_i}{2S}} \alpha_i \approx \sqrt{2S} \alpha_i . \quad (2)$$

By using the $1/S$ expansion, we approximately obtain the free bosonic Hamiltonian in the leading order in $1/S$:

$$H = \sum_{\mathbf{k}} (\omega(\mathbf{k}) - \mu) \alpha_{\mathbf{k}}^\dagger \alpha_{\mathbf{k}} , \quad (3)$$

where

$$\epsilon(\mathbf{k}) = \frac{1}{2} \sum_{\mathbf{l}} J_{\mathbf{l}} \cos \mathbf{k} \cdot \mathbf{l} , \quad \omega(\mathbf{k}) = 2S (\epsilon(\mathbf{k}) - \epsilon_{min}) , \quad (4)$$

$$\mu = 2S (\epsilon(0) - \epsilon_{min}) - (H + 2S K) ,$$

and ϵ_{min} is the minimum of $\epsilon(\mathbf{k})$. The conservation law of the angular momentum along the z direction leads to that of the total magnon number. For any exchange interactions, a sufficiently large H or K can induce the gap ($\mu \leq 0$) in the magnon dispersion. If $\mu \leq 0$, the ferromagnetic phase is stable (with few exceptions for finite S). Throughout this paper, we focus on this free bosonic Hamiltonian.

We have seen that frustration can induce a nontrivial dispersion relation. The nontrivial dispersion relation can lead to the nontrivial group velocity not parallel to the phase velocity, where the group velocity is given by

$$\mathbf{v}_g(\mathbf{k}) = \nabla_{\mathbf{k}} \omega(\mathbf{k}) . \quad (5)$$

Physically, the group velocity carry the angular momentum as reviewed in Appendix. Not the phase- but the group velocity determines the traveling direction of spinwave. Next, let us study the various dispersion relations in the concrete models.

2.1 simple case

First, we consider the simple Heisenberg model on the cubic lattice with the nearest neighbor ferromagnetic coupling $J < 0$ as shown in Fig. 1. $\epsilon(\mathbf{k})$ is given by

$$\epsilon_s(\mathbf{k}) = J(\cos k^a + \cos k^b + \cos k^c) ,$$

$$\epsilon_{smin} = \epsilon_s(\mathbf{k}_0 = (0, 0, 0)) = 3J . \quad (6)$$

This leads to the dispersion relation $\omega_s(\mathbf{k})$ shown in Fig. 2. In the long wavelength limit $|\mathbf{k}| \to 0$, this dispersion relation becomes isotropic and is given by

$$\omega_s(\mathbf{k}) \approx \frac{\mathbf{k}^2}{2m_s} , \quad m_s = -\frac{1}{J} . \quad (7)$$

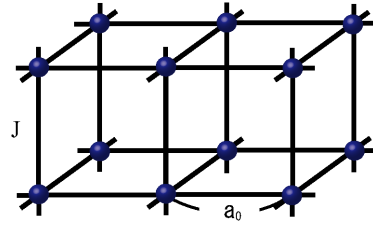In the following discussion, we use this model as the concrete example of 'usual' ferromagnets.

Fig. 1. (Color online) The cubic lattice with the nearest neighbor Heisenberg exchange coupling J. The dots represent spins. The lattice constant $a_0 = 1$ is assumed.
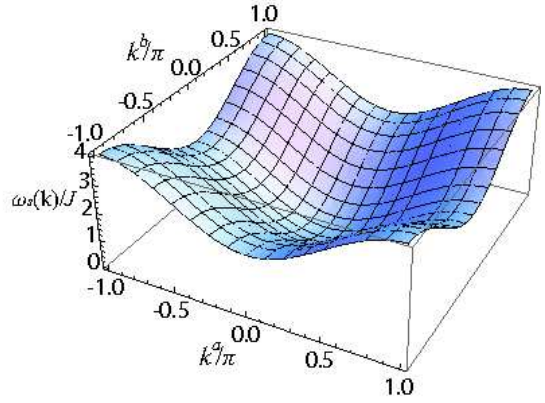

Fig. 2. (Color online) The dispersion relation $\omega_s(\mathbf{k})$ of the trivial ferromagnet on the cubic lattice for $J < 0$. $k^c = 0$ is assumed.

2.2 J_1-J_2 chain

Next, let us discuss the dispersion relation of the 1D J_1-J_2 chains with the ferromagnetic interchain coupling $J_3 < 0$ on the cubic lattice[16–18, 28] as illustrated in Fig. 3:

$$\epsilon_1(q) = J_1 \cos q^c + J_2 \cos 2q^c + J_3 (\cos q^a + \cos q^b)$$

$$= 2J_2 (\cos q^c + \frac{J_1}{4J_2})^2 - \frac{J_1^2}{8J_2} - J_2 + J_3 (\cos q^a + \cos q^b) , \quad (8)$$

where the J_1-J_2 chains are assumed to lie parallel to the c axis. For $|J_1|/J_2 \leq 4$, $J_2 > 0$ and $J_3 < 0$, the dispersion relation has two minima at $\mathbf{Q} = (0, 0, \pm Q^c)$ where

$$\cos Q^c = -\frac{J_1}{4J_2} , \quad \epsilon_{min} = -\frac{J_1^2}{8J_2} - J_2 + 2J_3 . \quad (9)$$

The dispersion relation $\omega_1(\mathbf{k})$ is graphically shown in Figs. 4,5.
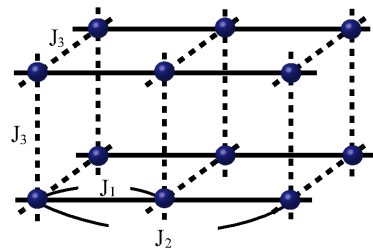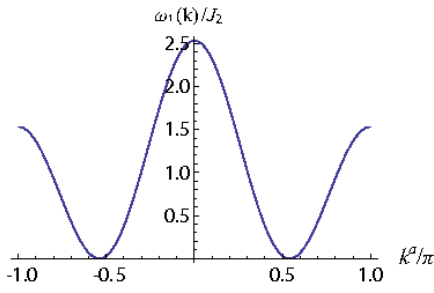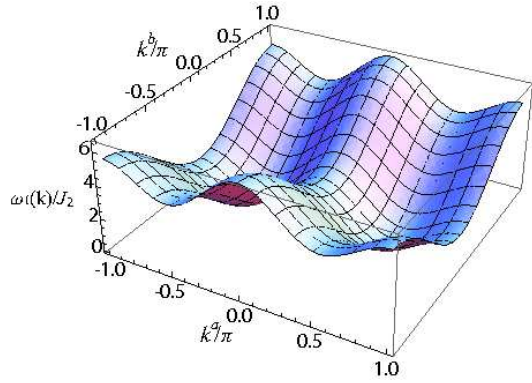

Fig. 3. (Color online) J_1-J_2 chains with the interchain coupling J_3 on the cubic lattice. The dots represent spins.

Fig. 4. (Color online) The dispersion relation $\omega_1(\mathbf{k})$ of the 1D J_1-J_2 chain for $J_1/J_2 = 0.5$ and $J_2 > 0$. $J_3 = 0$ or $k^b = k^c = 0$ is assumed.
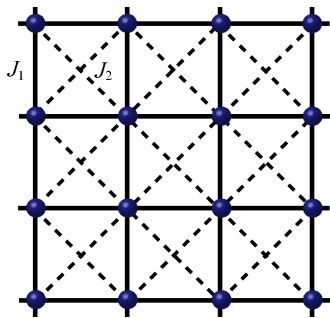

Fig. 5. (Color online) The dispersion relation $\omega_1(\mathbf{k})$ of the 1D J_1-J_2 chains on the cubic lattice for $J_1/J_2 = 0.5$, $J_2 > 0$ and $J_3/J_2 = -2$. $k^c = 0$ is assumed.
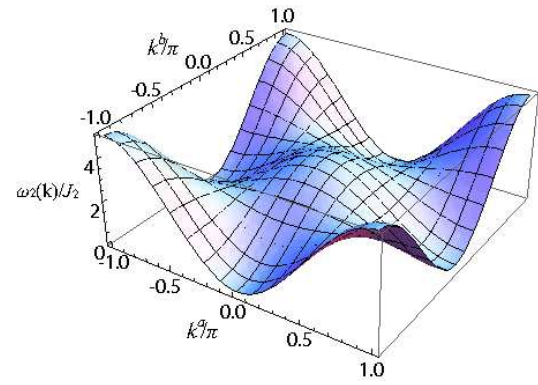
2.3 J_1-J_2 model on the square lattice

The fully saturated phase in J_1-J_2 models on the square lattice (see Fig. 6) also has the nontrivial dispersion relation since

$$\epsilon_2(q) = J_1(\cos q^a + \cos q^b) + J_2(\cos(q^a + q^b) + \cos(q^a - q^b)). \tag{10}$$

For $-2 < J_1/J_2 < 2$ and $J_2 > 0$, the dispersion relation $\omega_2(\mathbf{k})$ has two minima at $\mathbf{Q}_1^{(2)} = (0, \pi)$ and $\mathbf{Q}_2^{(2)} = (\pi, 0)$ as shown in Fig. 7.


Fig. 6. (Color online) Two dimensional square lattice. The dots represent spins connected by the nearest neighbor Heisenberg exchange coupling J_1 and the next nearest neighbor coupling J_2.

Fig. 7. (Color online) The dispersion relation $\omega_2(\mathbf{k})$ for $J_1/J_2 = -0.5$ and $J_2 > 0$ in the J_1-J_2 model on the two-dimensional square lattice.

3. The Snell's low in the ferromagnets junction system

In this section, we study the relation of the angles of incident-, reflected- and transmission- spinwaves passing through a boundary (the Snell's law). Let us consider a ferromagnet/ferromagnet junction system whose boundary plane is flat and is perpendicular to c direction. On the boundary the proximity effect may induce a spin-exchange coupling between spins on the boundary surface. This proximity effect leads to transmission of incident spinwave. The Schematic figure of the reflection and refraction process in the usual ferromagnets junction system is shown in Fig. 8.
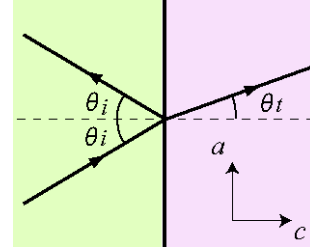
Fig. 8. (Color online) Schematic figure of refraction and reflection process of spinwave in the usual ferromagnets junction system in the case of $k_b = 0$. The travelling direction of spinwave is determined by the group velocity.

In the case of light, the Snell's low is understood by the conservation low of the energy and that of the momentum parallel to the surface. The same context can be applied to determines the Snell's low of spinwave in the ferromagnets junction system if the following conditions are satisfied: (i) each ferromagnet and the exchange interactions on the boundary have a rotational symmetry around the z direction in spin space; (ii) each magnet have a translational symmetry; (iii) the lattice vectors in the a-b plane are same in both magnets.[30] The assumption (i) leads to the conservation of a total number of magnons, and (ii),(iii) lead to the conservation law of a total momentum of the a and b direction. Of course, the total momentum conservation law of the c direction does not hold by the boundary effect.

Since the above discussion is too formal, let us study the Snell's low of spinwave by considering a simple model. For example, one of the simplest model satisfying these conditions is the junction system consists of ferromagnets with the

nearest neighbor interaction J (J') on the cubic lattice (see Sec. 2.1) with the same lattice constant. This setting is shown in Fig. 9. It is appropriate to consider the exchange interaction on the boundary as

$$\Delta \sum_{a,b} \mathbf{S}_{\mathbf{i}=(a,b,c_1)} \cdot \mathbf{S}_{\mathbf{j}=(a,b,c_2)} , \qquad (11)$$

where Δ is the strength of the exchange interaction on the boundary, and c_1 and c_2 are the c components of the boundary's position. This boundary condition satisfies the conditions (i) and (iii). As long as (i) and (iii) holds, the detailed boundary condition affect not the Snell's low of the spinwave but the transmission rate and the phase shift at the boundary. The explicit effect of the boundary condition will be discussed in Sec. 4.1.
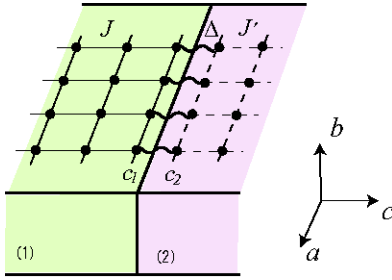
Fig. 9. (Color online) Junction system of the usual ferromagnets on the cubic lattice. The dots represent spins. The exchange coupling Δ due to the quantum proximity effect is considered on the boundary.

Let us consider the case that the incident spinwave in the medium (1) has the momentum $\mathbf{k}_i$. The incident spinwave arriving at the boundary is divided into the transmission wave with $\mathbf{k}_t$ and the reflected wave with $\mathbf{k}_r$.

The assumption (i) leads to the energy conservation

$$\Omega = \omega_{(1)}(\mathbf{k}_i) - \mu_{(1)} = \omega_{(1)}(\mathbf{k}_r) - \mu_{(1)} = \omega_{(2)}(\mathbf{k}_t) - \mu_{(2)} . \quad (12)$$

As discussed in Appendix, the group velocity corresponds to the current of magnons. Hence, it is appropriate in our setting to assume that the group velocity of the transmission (reflection) wave has the same (opposite) direction along the c axis as that of the incident wave. Namely,

$$v_g^{(1)c}(\mathbf{k}_i)/|v_g^{(1)c}(\mathbf{k}_i)| = -v_g^{(1)c}(\mathbf{k}_r)/|v_g^{(1)c}(\mathbf{k}_r)| = v_g^{(2)c}(\mathbf{k}_r)/|v_g^{(2)c}(\mathbf{k}_r)| . \quad (13)$$

In addition, the momentum conservation law parallel to the boundary leads to

$$k_i^a = k_r^a = k_t^a , \quad k_i^b = k_r^b = k_t^b . \qquad (14)$$

These equation determines the $\mathbf{k}_r$ and $\mathbf{k}_t$, which leads to the Snell's law. In general, the transimission- and reflected spinwaves determined by eqs. (12), (13), (14) are not necessarily single valued.

Finally, let us discuss the general relation in the reflection and refraction process of spinwaves if the condition (i) (ii) (ii) are satisfied. If we label spinwaves as $\mathbf{k}^{(1)}, \mathbf{k}^{(2)}, \mathbf{k}^{(3)} \cdots$, this

reflection and refraction process can be written as the ket

$$(A\alpha_{\mathbf{k}_i}^{(1)\dagger} + \sum_j B_j \alpha_{\mathbf{k}_r^{(j)}}^{(1)\dagger} + \sum_j C_j \alpha_{\mathbf{k}_t^{(j)}}^{(2)\dagger})|0\rangle , \qquad (15)$$

where $|0\rangle$ is vacuum state and $\alpha^{(1,2)}(\mathbf{k})$ approaches the free boson in the media (1,2) with the momentum $\mathbf{k}$ away from the boundary. By considering the cuboid discussed in Appendix, we obtain the magnon number conservation low:

$$|A|^2 v_g^{(1)c}(\mathbf{k}_i) + \sum_j |B_j|^2 v_g^{(1)c}(\mathbf{k}_r^{(j)}) = \sum_j |C_j|^2 v_g^{(2)c}(\mathbf{k}_t^{(j)}) . \quad (16)$$

The concrete expression of B and C will be discussed in Sec. 4.1 by explicitly considering the boundary condition.

Since we have discussed the procedure to find the Snell's low, let us discuss the detailed properties of the Snell's low in the various ferromagnets junction system.

3.1 usual case

First, let us further study the Snell's low of the junction system of the usual ferromagnets on the cubic lattice with the nearest neighbor exchange coupling $J < 0$ ($J' < 0$) as shown in Fig. 9. The dispersion relation on each ferromagnet is respectively given by

$$\omega_{s1}(\mathbf{k}) = 2S \left(J(\cos k^a + \cos k^b + \cos k^c) - 3J \right) , \qquad (17a)$$

$$\omega_{s2}(\mathbf{k}) = 2S \left(J'(\cos k^a + \cos k^b + \cos k^c) - 3J' \right), \qquad (17b)$$

where $\mu_1 < 0$ and $\mu_2 < 0$. $\mu_{1,2}$ are given in eq. (4). In these usual ferromagnets, the group velocity $\mathbf{v}_g$, which carry the angular momentum, is given by

$$\mathbf{v}_g^{(s1)}(\mathbf{k}) = \nabla_{\mathbf{k}}\epsilon(\mathbf{k}) = -2S J(\sin k^a, \sin k^b, \sin k^c) , \qquad (18a)$$

$$\mathbf{v}_g^{(s2)}(\mathbf{k}) = -2S J'(\sin k^a, \sin k^b, \sin k^c) . \qquad (18b)$$

The sign of the group velocity $v_g^{(s1)a,b,c}$ is always the same as the sign of $k^{a,b,c}$ for $-\pi < k^{a,b,c} < \pi$. If we consider the incident wave with wavevector $\mathbf{k}_i$, the reflected wave with $\mathbf{k}_r$ and the transmission wave with $\mathbf{k}_t$ are uniquely determined by eqs. (12), (13), (14). We obtain

$$\cos k_t^c = \frac{1}{J'}(J \cos k_i^c + (J - J')(\cos k_i^a + \cos k_i^b) - 3(J - J')$$
$$+ \frac{\mu_2 - \mu_1}{2S}) , \qquad (19)$$

where $k_t^c/|k_t^c| = k_i^c/|k_i^c|$ for $-\pi < k_t^c < \pi$. This implies that, the larger the chemical potential μ_2 is, the slower the group velocity of the transmitted wave becomes. Since $\mu_{1,2}$ depends on the external magnetic field H, we explicitly see that the propagation of the spinwave can be controlled by the external magnetic field as discussed in Refs. 2–5.

If we consider the long-wavelength limit $|\mathbf{k}| \to 0$, the Snell's low in this system becomes simple. The group velocity is proportional to the phase velocity in the long wavelength limit $\mathbf{k} \to 0$. Up to the order of $\mathbf{k}^2$, the refractive index is given by

$$n = \frac{\sin \theta_i}{\sin \theta_t} = \frac{|\mathbf{k}_t|}{|\mathbf{k}_c|} = \sqrt{\frac{\Omega - J' + \mu_2}{\Omega - J + \mu_1}} , \qquad (20)$$

where θ_i (θ_t) is the angle of incidence (refraction) as shown in Fig. 8, and Ω is given in eq. (12). Eq. (20) is exactly the same

as the result obtained in Ref. 3.

3.2 J_1-J_2 chains parallel to the boundary

Next, let us consider the reflection and refraction process of spinwaves in the usual ferromagnet/frustrated ferromagnet junction system. In this subsection, as a concrete example of frustrated ferromagnets, we consider the J_1-J_2 chains on the cubic lattice discussed in Sec. 2.2. We assume that the J_1-J_2 chains are parallel to the boundary plane as shown in Fig. 10.

The dispersion relation and the group velocity in the J_1-J_2 chains on the cubic lattice are given by

$$\omega_{11}(\mathbf{k}) = 2S(J_1 \cos k^a + J_2 \cos 2k^a$$
$$+ J'(\cos k^b + \cos k^c - 2) - \frac{J_1^2}{8J_2} - J_2),$$
$$\mathbf{v}_g^{(11)}(\mathbf{k}) = -2S(J_1 \sin k^a + 2J_2 \sin 2k^a, J' \sin k^b, J' \sin k^c).$$
$$(21)$$

If we consider the incident wave of the wavevector $\mathbf{k}_i$ in the usual ferromagnet, the transmission wave of $\mathbf{k}_t$ is uniquely given by eqs. (12), (13), (14):

$$\cos k_t^c = \frac{1}{J'}(\frac{\Omega + \mu_2}{2S} - J_1 \cos k_i^a - J_2 \cos 2k_i^a$$
$$- J'(\cos k_i^b - 1) - \frac{J_1^2}{8J_2} - J_2). \qquad (22)$$

where $k_t^c/|k_t^c| = k_i^c/|k_i^c|$ for $-\pi < k_t^c < \pi$. As discussed in Sec 2.2, if $-4 < J_1/|J_2| < 4$ and $J_2 > 0$, the equation $J_1 \cos k_i^a + J_2 \cos 2k_i^a$ has two minima at $\cos k_a^i = -\frac{J_1}{4J_2}$. If $\cos k_i^a > -\frac{J_1}{4J_2}$, the a component of the group velocities of incident spinwave is of the opposite sign to that of the transmission spinwave: the refractive index becomes negative[29] as shown in Fig. 11.
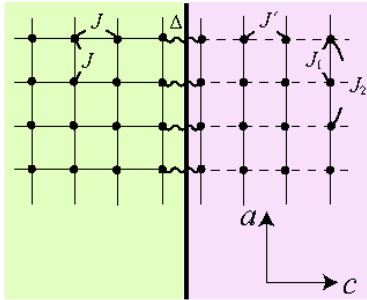
Fig. 10. (Color online) Junction system of the usual ferromagnet (left, green) /ferromagnet consisting of J_1-J_2 chains which lie along the a axis (right, red). The exchange coupling Δ due to the quantum proximity effect is considered on the boundary.

3.3 J_1-J_2 chains perpendicular to the boundary

In this subsection, we study the case that the frustrated ferromagnet consists of J_1-J_2 chains is perpendicular to the boundary plane as show in Fig. 12. The dispersion relation in
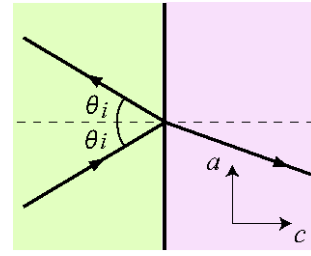
Fig. 11. (Color online) Schematic figure of refraction and reflection process of spinwave in the case of the negative refraction index and $\mathbf{k}_b = 0$. The travelling direction of spinwave is determined by the group velocity.

the J_1-J_2 chains is given by

$$\omega_{12}(\mathbf{k}) = 2S(J_1 \cos k^c + J_2 \cos 2k^c$$
$$+ J'(\cos k^a + \cos k^b - 2) + \frac{J_1^2}{8J_2} + J_2), \qquad (23)$$

If the spinwave of the wavevector $\mathbf{k}_i$ in the usual ferromagnet is injected into the J_1-J_2 chains, the transmission waves of $\mathbf{k}_t^{(\pm)}$ are given by eqs. (12), (13), (14):

$$\cos k_t^{(\pm)c} = \pm \sqrt{\frac{\Omega - 2S J'(\cos k^a + \cos k^b - 2) + \mu_2}{4S J_2} - \frac{J_1}{4J_2}}, \qquad (24)$$

where $-\frac{J_1 \sin k_t^{(\pm)c} + 2J_2 \sin 2k_t^{(\pm)c}}{|J_1 \sin k_t^{(\pm)c} + 2J_2 \sin 2k_t^{(\pm)c}|} = \frac{k_i^c}{|k_i^c|}$. This result implies that in some cases the two species of the transmission spinwaves appear as shown in Fig. 13.
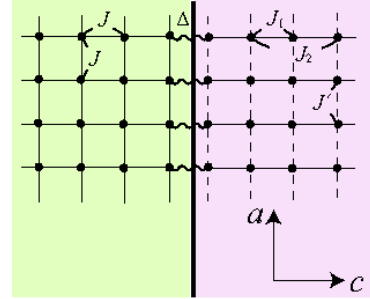

Fig. 12. (Color online) Junction system of the usual ferromagnet (left, green) /ferromagnet consisting of J_1-J_2 chains which lie along the c axis (right, red). The exchange coupling Δ due to the quantum proximity effect is considered on the boundary.

3.4 J_1-J_2 model on the square lattice

Finally, let us briefly study the fully polarized phase in the J_1-J_2 model on the square lattice as an example of frustrated ferromagnets. We study the 2-dimensional junction system of the a-b plane as shown in Fig. 14. The dispersion relation in the J_1-J_2 model is given by eq. (10). In the case that $-2 < J_1/J_2 < 2$ and $J_2 > 0$, by injecting the spinwave of $\mathbf{k}_i$ from the usual ferromagnet, the transmission spinwave of $\mathbf{k}_t$ in the
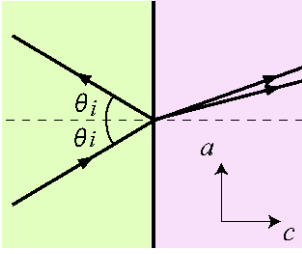
Fig. 13. (Color online) Schematic figure of refraction and reflection process of spinwave in the case of $\mathbf{k}_b = 0$. The incident spinwave can split into two transmission waves. The travelling direction of spinwave is determined by the group velocity.

J_1-J_2 model is given by

$$k_t^a = k_i^a ,$$

$$\cos k_t^b = \frac{1}{J_1 + 2J_2 \cos k_i^a} \left(\frac{\Omega + \mu_2}{2S} - J_1 \cos k_i^a - 2J_2 \right) , \tag{25}$$

where $-\frac{J_1 \sin q_b + 2J_2 \cos q_a \sin q_b}{|J_1 \sin q_b + 2J_2 \cos q_a \sin q_b|} = -\frac{\sin k_t^b}{|\sin k_i^b|}$. As well as the junction system discussed in Sec. 3.2, the negative refraction can be realized as shown in Fig. 11.
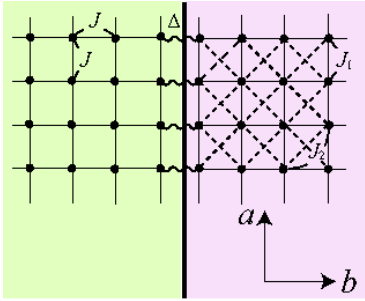
Fig. 14. (Color online) Junction system of the usual ferromagnet (left, green) /ferromagnet consisting of J_1-J_2 model on the square lattice (right, red). The exchange coupling Δ due to the quantum proximity effect is considered on the boundary.

4. Reflectance and Transmittance

We have studied the relation between the refraction angle of transmission spinwaves and the angle of incident spinwave, namely, the Snell's law of spinwaves in ferromagnets junction system. In this section, let us discuss the reflectance and transmittance of spinwaves. Although in this section we focus on the concrete models discussed in Secs. 3.1, 3.2, 3.3, our approach will be easily applied to the other frustrated ferromagnets junction systems satisfying the conditions (i), (ii), (iii) discussed in Sec. 3.

4.1 usual case

Let us study the reflectance and transmittance of spinwaves in the usual ferromagnet/ferromagnet junction system on the cubic lattice with the boundary condition (11) (see Fig. 9). As discussed in Sec. 3, the reflection and refraction process by

the incident spinwave of $\mathbf{k}_i$ may be written as the ket

$$|RR_1\rangle = \sum_{a,b} e^{i(k_i^a a + k_i^b b)} \left(A \sum_{c=-\infty}^{0} e^{ik_i^c c} \alpha_{a,b,c}^\dagger + B \sum_{c=-\infty}^{0} e^{-ik_r^c c} \alpha_{a,b,c}^\dagger \right.$$
$$\left. + C \sum_{c=1}^{\infty} e^{ik_t^c (c-1)} \alpha_{a,b,c}^\dagger \right) |0\rangle , \tag{26}$$

where we assume that the boundary is located between $c = 0$ and 1, and k_t^c is given by eq. (19). Then, A, B, C is determined by the Schröedinger equation:

$$H|RR_1\rangle = \Omega|RR_1\rangle . \tag{27}$$

Obviously, (27) is satisfied at any lattice cite except at $c = 0, 1$. Hence, we consider

$$\langle 0|\alpha_{a,b,c=0} H|RR_1\rangle = \Omega(A + B)e^{i(k_i^a a + k_i^b b)} ,$$
$$\langle 0|\alpha_{a,b,c=1} H|RR_1\rangle = \Omega C e^{i(k_i^a a + k_i^b b)} . \tag{28}$$

Equivalently,

$$(\epsilon_1(k_i^c) - Je^{-ik_i^c})A + (\epsilon_1(k_i^c) - Je^{ik_i^c})B = \Delta C ,$$
$$\Delta(A + B) = (\epsilon_2(k_t^c) - J'e^{ik_t^c})C , \tag{29}$$

where

$$\epsilon_1(k_i^c) = 2J(\cos k_i^c - 1) - J + \Delta ,$$
$$\epsilon_2(k_t^c) = 2J'(\cos k_t^c - 1) - J' + \Delta . \tag{30}$$

Hence, we obtain

$$\frac{B}{A} = -\frac{(\epsilon_1(k_i^c) - Je^{-ik_i^c})(\epsilon_2(k_t^c) - J'e^{ik_t^c}) - \Delta^2}{(\epsilon_1(k_i^c) - Je^{ik_i^c})(\epsilon_2(k_t^c) - J'e^{ik_t^c}) - \Delta^2} ,$$
$$\frac{C}{A} = -\Delta \frac{2iJ \sin k_i^c}{(\epsilon_1(k_i^c) - Je^{ik_i^c})(\epsilon_2(k_t^c) - J'e^{ik_t^c}) - \Delta^2} . \tag{31}$$

By using the relation

$$|(\epsilon_1(k_i^c) - Je^{-ik_i^c})(\epsilon_2(k_t^c) - J'e^{ik_t^c}) - \Delta^2|^2$$
$$- |(\epsilon_1(k_i^c) - Je^{ik_i^c})(\epsilon_2(k_t^c) - J'e^{ik_t^c}) - \Delta^2|^2 \tag{32}$$
$$= 4JJ'\Delta^2 \sin k_i^c \sin k_t^c = \Delta^2 v_g^{(s1)}(\mathbf{k}_i) v_g^{(s2)}(\mathbf{k}_t)/S^2 ,$$

the magnon-number-conservation law given by eq. (16) is easily confirmed as

$$v_g^{(s1)c}(\mathbf{k}_i)\left(1 - \left|\frac{B}{A}\right|^2\right) = v_g^{(s2)c}(\mathbf{k}_t)\left|\frac{C}{A}\right|^2 , \tag{33}$$

regardless of any $\mathbf{k}_i$ and $\mathbf{k}_t$. From eq. (31), we explicitly find the dependence of the transmittance and the phase shift of spinwave on the boundary condition Δ. For $\Delta \to 0$, the transmittance leads $|C/A|^2 = O(\Delta^2/J'^2)$.

4.2 J_1-J_2 chains parallel to the boundary

Next, let us study the case that one side of the ferromagnets-junction system consists of the J_1-J_2 chains parallel to the boundary as shown in Fig. 10. The reflection and refraction

process is described by the following ket:

$$|RR_2\rangle = \sum_{a,b} e^{i(k_i^a a + k_i^b b)}(A \sum_{c=-\infty}^{0} e^{ik_i^c c}\alpha_{a,b,c}^\dagger + B \sum_{c=-\infty}^{0} e^{-ik_r^c c}\alpha_{a,b,c}^\dagger$$

$$+ C \sum_{c=1}^{\infty} e^{ik_t^c(c-1)}\alpha_{a,b,c}^\dagger)|0\rangle ,$$

$$(34)$$

where k_t^c is given by eq. (22). The procedure to determine B, C is exactly the same as that of the previous section. Hence, B and C are given by (31).

4.3 J_1-J_2 chains perpendicular to the boundary

Finally, we consider the J_1-J_2 chains perpendicular to the boundary as a example of frustrated ferromagnets as show in Fig. 12. In this case, the ket is given by

$$|RR_3\rangle = \sum_{a,b} e^{i(k_i^a a + k_i^b b)}(A \sum_{c=-\infty}^{0} e^{ik_i^c c}\alpha_{a,b,c}^\dagger + B \sum_{c=-\infty}^{0} e^{-ik_r^c c}\alpha_{a,b,c}^\dagger$$

$$+ C_1 \sum_{c=1}^{\infty} e^{ik_t^{(+)c}(c-1)}\alpha_{a,b,c}^\dagger + C_2 \sum_{c=1}^{\infty} e^{ik_t^{(-)c}(c-1)}\alpha_{a,b,c}^\dagger)|0\rangle ,$$

$$(35)$$

where $k_t^{(\pm)c}$ are given by eq. (24). We see that (27) is satisfied at any lattice cite except at $c = 0, 1, 2$. Hence, we consider

$$\langle 0|\alpha_{a,b,c=0}H|RR_1\rangle = \Omega(A + B)e^{i(k_i^a a + k_i^b b)} ,$$

$$\langle 0|\alpha_{a,b,c=1}H|RR_1\rangle = \Omega(C_1 + C_2)e^{i(k_i^a a + k_i^b b)} , \qquad (36)$$

$$\langle 0|\alpha_{a,b,c=2}H|RR_1\rangle = \Omega(C_1 e^{ik_t^{(+)c}} + C_2 e^{ik_t^{(-)c}})e^{i(k_i^a a + k_i^b b)} .$$

Equivalently,

$$(\epsilon_1'(k_i^c) - Je^{-ik_i^c})A + (\epsilon_1'(k_i^c) - Je^{ik_i^c})B = \Delta(C_1 + C_2) ,$$

$$(\epsilon_2'(k_t^{(+)c}) - J_1 e^{ik_t^{(+)c}} - J_2 e^{i2k_t^{(+)c}})C_1$$

$$+ (\epsilon_2'(k_t^{(+)c}) - J_1 e^{ik_t^{(-)c}} - J_2 e^{i2k_t^{(-)c}})C_2 = \Delta(A + B) , \qquad (37)$$

$$(\epsilon_3'(k_t^{(+)c}) - 2J_1 \cos k_t^{(+)c} - J_2 e^{i2k_t^{(+)c}})C_1 e^{ik_t^{(+)c}}$$

$$+ (\epsilon_3'(k_t^{(+)c}) - 2J_1 \cos k_t^{(-)c} - J_2 e^{i2k_t^{(-)c}})C_2 e^{ik_t^{(-)c}} = 0 ,$$

where

$$\epsilon_1'(k_i^c) = 2J(\cos k_i^c - 1) - J + \Delta$$

$$\epsilon_2'(k_t^{(+)c}) = \epsilon_2'(k_t^{(-)c}) = 2(J_1 \cos k_t^{(+)c} + J_2 \cos 2k_t^{(+)c} + \frac{J_1^2}{8J_2})$$

$$- J_1 + J_2 + \Delta ,$$

$$\epsilon_3'(k_t^{(+)c}) = \epsilon_2'(k_t^{(+)c}) + J_1 - \Delta .$$

$$(38)$$

$B/A, C_1/A, C_2/A$ are given by solving these equations. We numerically confirmed the following magnon number conservation law in the various concrete parameters:

$$v_g^{(s1)c}(\mathbf{k}_i)(1-|\frac{B}{A}|^2) = v_g^{(12)c}(\mathbf{k}_t^{(+)})|\frac{C_1}{A}|^2 + v_g^{(12)c}(\mathbf{k}_t^{(-)})|\frac{C_2}{A}|^2 , \quad (39)$$

where $v_g^{(12)c}(\mathbf{k}) = -2S(J_1 \sin k^c + J_2 \sin 2k^c)$.

5.   Conclusion

We studied the reflection and refraction process of spinwaves in the ferromagnet/frustrated ferromagnet junction system whose ferromagnets are described by the spin Hamiltonian with the generic Heisenberg-exchange coupling, the uniaxial anisotropic interaction and the external magnetic field. By using the Holstein-Primakoff spinwave expansion and taking the large S limit, the ferromagnet phase is given by the free bosonic Hamiltonian, which describes the dynamics of magnons. If frustration exists, the dispersion relation of magnons (spinwave) can become nontrivial.

In Sec. 3, we discussed the Snell's low of the spinwave in the case that the following condition is satisfied in the junction system: (i) each ferromagnet and the exchange interactions on the boundary have a rotational symmetry around the z direction in spin space; (ii) each magnet have a translational symmetry; (iii) the lattice vectors in the boundary plane are same in both magnets.[30] By studying the various junction systems satisfying these conditions, we found the nontrivial refraction process, e.g., the splitting of the spinwave and the negative refraction.

In Sec. 4, we studied the 'reflectance', 'transmittance' and 'phase shift at the boundary' in the concrete examples of the junction systems. Within the large S limit, we exactly obtain these quantities, which explicitly depend on the boundary condition. Throughout this paper our results does not need the long wavelength approximation.

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Appendix: Current of angular momentum

In this appendix, we briefly review the correspondence between the group velocity of spinwave and the current of angular momentum. We study the spin system described by the Hamiltonian (1). Now, the Hamiltonian (1) commutes with the total spin $S_{\text{tot}}^z = \sum_l S_{\mathbf{j}}^z$. Hence, S_{tot}^z is the conserved quantity: the total magnon number $\sum_l a_l^\dagger a_l$ conserves. This implies that the conserved current of magnons (related to the U(1) symmetry) exists.

The magnon current is understood by studying in- and outflow of the magnon number in a given region. Let us consider the cuboid C_1 of which two face-to-face surfaces S_1 and S_2 are infinitely large as shown in Fig. A·1.
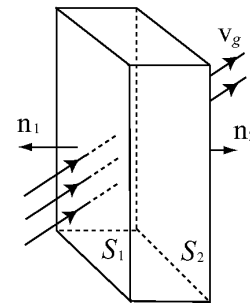

Fig. A·1. Magnon current of the group velocity v_g passes through the cuboid C_1.

The spatially-averaged flow of magnons is determined by

the current passing through S_1 and S_2. By using the relation

$$[\alpha_\mathbf{i}^\dagger \alpha_\mathbf{i} + \alpha_\mathbf{j}^\dagger \alpha_\mathbf{j}, \mathbf{S_i} \cdot \mathbf{S_j}] = 0 , \tag{A·1}$$

we obtain

$$i\frac{d(\sum_{l \in C_1} \alpha_\mathbf{l}^\dagger \alpha_\mathbf{l})}{dt} = [\sum_{l \in C_1} \alpha_\mathbf{l}^\dagger \alpha_\mathbf{l}, H] = S \sum_{i \in C_1, \ j \notin C_1} J_{\mathbf{ij}}(\alpha_\mathbf{i}^\dagger \alpha_\mathbf{j} - \alpha_\mathbf{i} \alpha_\mathbf{j}^\dagger) , \tag{A·2}$$

where we use eq. (2). We assume that $J_{\mathbf{ij}} = J_\mathbf{l}$ is short range so that $J_\mathbf{l} = 0$ for a large $|\mathbf{l}|$, and that S_1 is sufficiently separated from S_2. Then, if we consider the state $|\mathbf{k}\rangle = \alpha_\mathbf{k}^\dagger|0\rangle$, we obtain

$$\langle\mathbf{k}|\frac{d(\sum_{l \in C_1} \alpha_\mathbf{l}^\dagger \alpha_\mathbf{l})}{dt}|\mathbf{k}\rangle = \frac{2S}{N} \sum_{\mathbf{i} \in C_1, \ \mathbf{j} \notin C_1} J_{\mathbf{ij}} \sin \mathbf{k} \cdot (\mathbf{i} - \mathbf{j})$$

$$= \frac{1}{N}(A_{S_1}\mathbf{n}_1 \cdot \mathbf{v}_g(\mathbf{k}) + A_{S_2}\mathbf{n}_2 \cdot \mathbf{v}_g(\mathbf{k})) , \tag{A·3}$$

where A_{S_i} is the area of S_i and

$$\mathbf{v}_g(\mathbf{k}) \cdot \mathbf{n} = 2S \frac{\partial \epsilon(\mathbf{k})}{\partial \mathbf{k}} \cdot \mathbf{n} = -2S \sum_\mathbf{l} J_\mathbf{l} \sin(\mathbf{k} \cdot \mathbf{l})\mathbf{l} \cdot \mathbf{n} . \tag{A·4}$$

These equations imply that the group velocity can be viewed as the magnon current which carries the angular momentum. Even if there is a superposition of various spinwaves such as $\sum_l A_l a_{\mathbf{k}_l}^\dagger|0\rangle$, the time-averaged magnon current is simply summed as

$$\bar{\mathbf{v}}_g^{\text{tot}} = \sum_l |A_l|^2 \mathbf{v}_g(\mathbf{k}_l) . \tag{A·5}$$

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